

Practicality of Quantum Computing & AI

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ABSTRACT

The main purpose of this paper is to examine some (potential) applications of quantum computation in AI and to review the interplay between quantum theory and AI. For the readers who are not familiar with quantum computation, a brief introduction to it is provided, and a famous but simple quantum algorithm is introduced so that they can appreciate the power of quantum computation. Also, a (quite personal) survey of quantum computation is presented in order to give the readers a (unbalanced) panorama of the field. The author hopes that this paper will be a useful map for AI researchers who are going to explore further and deeper connections between AI and quantum computation as well as quantum theory although some parts of the map are very rough and other parts are empty, and waiting for the readers to fill in.

Keywords — Quantum Computation, Artificial Intelligence, Autonomy, Cryptography, Privacy Protection

I. INTRODUCTION

Quantum theory is without any doubt one of the greatest scientific achievements of the 20th century. It provides a uniform framework for the construction of various modern physical theories. After more than 50 years from its inception, quantum theory married with computer science, another great intellectual triumph of the 20th century and the new subject of quantum computation was born.

Quantum computers were first envisaged by Nobel Laureate physicist Feynman [47] in 1982. He conceived that no classical computer could simulate certain quantum phenomena without an exponential slowdown, and so realized that quantum mechanical effects should offer something genuinely new to computation. In 1985, Feynman's ideas were elaborated and formalized by Deutsch in a seminal paper [30] where a quantum Turing machine was described. In particular, Deutsch introduced the technique of quantum parallelism based on the superposition principle in quantum mechanics by which a quantum Turing machine can encode many inputs on the same tape and perform a calculation on all the inputs simultaneously. Furthermore, he proposed that quantum computers might be able to perform certain types of computation that classical computers can only perform very inefficiently.

One of the most striking advances was made by Shor [31] in 1994. By exploring the power of quantum parallelism, he discovered a polynomial-time algorithm on quantum computers for prime factorization of which the best known algorithm on classical computers is exponential. In 1996, Grover [52] offered another killer application of quantum computation, and he found a quantum algorithm for searching a single item in an unsorted database in square root of the time it would take on a classical computer. Since database search and prime factorization are central problems in computer science and cryptography, respectively, and the quantum algorithms for them are much faster than the classical ones, Shor and Grover's works stimulated an intensive investigation

in quantum computation. Since then, quantum computation has been an extremely exciting and rapidly growing field of research.

The models of quantum computation have their ancestors from the studies of connections between physics and computation. In 1973, to understand the thermodynamics of classical computation Bennet [13] noted that a logically reversible operation does not need to dissipate any energy and found that a logically reversible Turing machine is a theoretical possibility. In 1980, Benioff [11] constructed a quantum mechanical model of a Turing machine. His construction is the first quantum mechanical description of computer, but it is not a real quantum computer because the machine may exist in an intrinsically quantum state between computation steps, but at the end of each computation step the tape of the machine always goes back to one of its classical states. The first truly quantum Turing machine was described by Deutsch [30] in 1985. In his machine, the tape is able to exist in quantum states too. This is different from Benioff's machine. A thorough exposition of the quantum Turing machine is given in [14].

In the realm of classical computation, finite automata and pushdown automata have been widely applied in the design and implementation of programming languages. Several quantum generalizations of finite and pushdown automata were introduced by Kondas and Watrous [23], Gudder [54], and Moore and Crutchfield [39] in the late 1990's. Their definitions of quantum automata differ mainly in where quantum measurements are allowed. For example, a quantum automaton introduced in [29] may be observed only after all input symbols have been read, whereas a quantum automaton in [33] is allowed to be observed after reading each symbol. The most general model of quantum finite automata was proposed independently by Bertoni, Mereghetti and Palano [15] and Ciamarra [25], and it admits any sequence of unitary transformations and measurements.

Recently, some applications of quantum automata have been found; for example, Nishimura and Yamakami [36] provided a direct application of quantum automata to interactive proof systems. But it seems not the case that quantum automata can be used in compiling of quantum programming languages. Since it revolutionized the very notion of computation, quantum computation forces us to reexamine various branches of computer science, and AI is not an exception. Roughly speaking, AI has two overall goals: (1) engineering goal – to develop intelligent machines; and (2) scientific goal – to understand intelligent behaviors of humans, animals and machines [55]. AI researchers mainly employ computing techniques to achieve both the engineering and scientific goals. Indeed, recently, McCarthy [8] even pointed out that “computational intelligence” is a more suitable name of the subject of AI to highlight the key role played by computers in AI. Naturally, the rapid development of quantum computation leads us to ask the question: how can this new computing technique help us in achieving the goals of AI. It seems obvious that quantum computation will largely contribute to the engineering goal of AI by applying it in various AI systems to speedup the computational process, but it is indeed very difficult to design quantum algorithms for solving certain AI problems that are more efficient than the existing classical algorithms for the same purpose.

At this moment, it is also not clear how quantum computation can be used in achieving the scientific goal of AI, and to the best of my knowledge there are no serious research pursuing this problem. Instead, it is surprising that quite a large amount of literature is devoted to applications of quantum theory in AI and vice versa, not through quantum computation. It can be observed from the existing works that due to its inherent probabilistic nature, quantum theory can be connected to numerical AI in a more spontaneous way than to logical AI.

The aim of this paper is two-fold: (1) to give AI researchers a brief introduction and a glimpse of the panorama of quantum computation; and (2) to examine connections between quantum computation, quantum theory and AI. The remainder of the paper is organized as follows: Section 2 is a tutorial of quantum computation for readers who are not familiar with quantum computation and quantum theory. Section 3 surveys some areas of quantum computation which the author is familiar with. Some potential applications of quantum computation in AI are considered in Section 4, and the interplay between quantum theory and AI is discussed in Section 5. A brief conclusion is drawn in Section 6.

II. POTENTIAL APPLICATIONS OF QUANTUM COMPUTATION

Quantum computation researchers hope to find more quantum algorithms demonstrating significant speedup over classical algorithms. They are looking for new problems suited to this purpose, and some AI problems seems to be good candidates. On the

other hand, the AI community believes that quantum computation shows significant potential for solutions to currently intractable problems. Indeed, 10 years ago, the “Trends and Controversies” of the July/August issue of the magazine *IEEE Intelligent Systems* was devoted to the possibility of combining quantum computation and AI [26]. Also, some quantum computation researchers were invited to present Tutorials at *IJCAI* conferences. To the best of my knowledge, however, not much progress has been made in this direction up to now. Perhaps, this is because not much effort has been expended, the majority of AI community may think that quantum computing technology is still in its infancy, and it is too early to consider how quantum computation can be used in AI. So, what we can do in this section is to explore some of possibilities of applying quantum computation in AI rather than to review the existing applications of quantum computation in AI.

A. Quantum Algorithms for Learning

Maybe the only area where quantum computation and AI have already met in a fruitful way is machine learning. There are several papers devoted to quantum generalization of computational learning theory. Their aim is to find some quantum algorithms that are more efficient than the existing classical algorithms for learning of certain classical objects, such as Boolean functions. This research is closely related to quantum complexity theory [14]. I am not an expert in this area, but fortunately a good survey of it already exists [18]. This survey is not new, but it is quite comprehensive. A dual topic is learning objects in the quantum world using mainly classical methods (together with quantum measurements).

B. Quantum Algorithms for Decision Problems

Many decision problems can be formulated in terms of decision trees. Farhi and Gutmann [42] showed that quantum algorithms based on Hamiltonian evolution can solve the decision problems represented by a class of decision trees exponentially faster than classical random walks. But this does not imply any advantage of quantum computation over classical computation for this class of problems because they can also be solved very quickly by other classical algorithms.

C. Quantum Search

Much of the early AI research was concerned with search techniques. This may be because on the one hand, many AI problems can be reduced to searching; for example, planning, scheduling, theorem proving and information retrieval, and on the other hand, computers can do these kinds of tasks much faster than humans. The Grover algorithm [52] shows that quantum computers can do it even faster than classical computers. Naturally, people expect that quantum computation will be widely used in AI to solve various search-related problems. It is believed that quantum searching

will be one of the first quantum computing techniques that play an important role in AI. In 1999, Hogg [51] discussed the problem of how quantum search algorithms can be applied in AI in detail. But up to now, 10 years later, few successful applications of quantum searching in AI have been reported.

D. Quantum Game Theory

Game theory is being used in AI progressively more and more, especially in multi-agent systems and distributed AI. Recently, quantum extensions of game theory have been proposed in a series of papers; for example, Eisert, Wilkens and Lewenstein [39] introduced quantization of nonzero sum games with two players, and Benjamin and Hayden [12] introduced quantum games with more than two players. Miakisz, Piotrowski and Ślaskowski [38] argued that quantum game theory [39] offers new tools for solutions of some problems in AI.

Other possibilities of applying quantum computation in AI include:

- Representing knowledge in the way of quantum superposition, and speeding up knowledge reasoning by quantum parallelism.
- Using quantum communication and distributed quantum computation in multi-agent systems; in particular, using entanglement for coordination.

III. INTERPLAY BETWEEN QUANTUM THEORY & AI

E. Semantic Analysis

Some similarities between the mathematical structure used by the AI community in semantic analysis of natural language and those employed in quantum mechanics were observed in [5]. But these similarities exposed in [5] seems very superficial, and they do not convince me to believe that a certain intrinsic connection exists between semantic analysis and quantum mechanics because it is not surprising that the same mathematical tools can be applied in unrelated domains, and indeed universal effectiveness is exactly one of the most important advantages of mathematics. On the other hand, however, observation of these similarities is still useful since by analogy it may provide hints as to how one can borrow some ideas from the well-established subject of quantum mechanics in semantic analysis or even more broadly in AI. Furthermore, if some semantic aspects of natural languages can be properly expressed in the framework of quantum theory, e.g. ambiguity by superposition, then the fact that quantum algorithms are especially suited to simulation of quantum systems suggests that quantum computation might considerably speedup natural language processing.

F. Entanglement of Words in Natural Languages

Nelson, McEvoy and Pointer [22] noticed that word associations in natural languages can display ‘spooky action at a distance behavior’. Bruza et al. [20] proposed a model of word associations in terms of tensor products so that ‘spooky activation at a distance’ can be described in a way similar to quantum entanglement.

We now turn to consider the inverse problem: how can some ideas developed in AI be used in quantum theory. The research on this problem can also be seen from another point of view. The current AI community is mainly devoted to develop computing techniques that implement intelligence for dealing with problems in the classical world. The research considered in the following subsections can be thought of as AI techniques that implement intelligence for coping with problems in the quantum world. In fact, the quantum counterparts of some basic AI problems such as learning and pattern recognition have been identified and intensively studied by physicists working in the fields of quantum information. It seems that AI researchers do not know much about this kind of work. I believe that AI researchers’ participation in understanding quantum information will accelerate the development of this area, and the methodologies and techniques developed by AI researchers will help quantum physicists.

G. Quantum Bayesian Networks

Statistical inference is at the heart of quantum theory due to the essential probabilistic nature of quantum systems. Bayesian methods have been widely used in statistical inference in the classical world. Recently, several versions of quantum Bayes rule have been derived in the physics literature; see for example [46].

Bayesian networks are graph models for representing and reasoning about probability information and widely used in AI. It is hoped that this kind of graph model can be adopted in reasoning about the behaviors of large systems in the quantum world. Tucci [38] introduced a quantum generalization of Bayesian networks in which complex amplitudes rather than (conditional) probabilities are assigned to its nodes and used it to calculate probabilities for some physical experiments. Pearl [28] introduced the notion of causal Bayesian networks which augments Bayesian networks with a set of local operations that specify how probability distributions behave with respect to external interventions. To provide a graph model of causality in the quantum world, Laskey [34] defined a notion of quantum causal networks where the local operations are represented by super-operators that are a popular mathematical formalism of the dynamics of open quantum systems.

H. Recognition and Discrimination of Quantum States and Quantum Operations

Pattern recognition is an important area of AI, and discrimination of objects can be seen as a special case of pattern recognition. However, only recognition and

discrimination of classical objects have been considered by AI researchers. In the last 20 years, a large amount of work on discrimination and recognition of quantum states and quantum operations has been conducted by physicists without knowing much about existing AI work.

Recently, discrimination of quantum operations has received considerable attention. The problem of discriminating (global) unitary transformations (quantum gates) was solved by Acín [4] and D'Ariano, Presti and Paris [29], and studies on discrimination of quantum measurements were initiated in [55]. The general case of (global) quantum operations represented by super-operators was considered in [10]. In particular, a complete characterization of perfect distinguishability of quantum operations was achieved in [36] by discovering a feasible necessary and sufficient condition under which an unknown quantum operation secretly chosen from a finite set of quantum operations can be identified perfectly and by designing an optimal protocol for such a discrimination with a minimal number of queries. A particularly interesting problem is discrimination of quantum operations acting on a multipartite quantum system by local operations and classical communication (LOCC for short). Surprisingly, it is proved in [33–35] that entanglement is unnecessary for this kind of discrimination of unitary operators although it had been believed that entanglement was necessary.

I. Learning of Quantum States and Quantum Operations

Quantum state tomography [20] can be seen as a kind of quantum learning. The scenario is as follows: There is a physical process that can produce a quantum state repeatedly. We prepare as many copies of the state as needed by applying this process. Our goal is to learn a description of the state from the measurement outcomes performed on these copies. A similar problem for quantum operations is known as quantum process tomography of which a theory was developed by Chuang and Nielsen [24] and Poyatos, Cirac and Zoller [21].

The studies of learning in the quantum world are still at the initial stage. Quantum generalizations of various sophisticated machine learning methods are entirely untouched. This presents a good opportunity to AI researchers because physicists may not be aware of these methods.

Other research arising from the interplay between quantum theory and AI include:

- Quantum neural networks, see for example [40].
- Quantum genetic algorithms, see for example [11].

There are many interesting topics for which a proper problem statement and an appropriate setting are still unknown. Here I only mention:

- Spatial reasoning in the quantum world.
- Constraint satisfaction of quantum states.

Certain interplay between quantum theory and AI has been examined in this section, but a much deeper connection between these two subjects may come from macroscopic quantum effects in the brain as is explored by Penrose [49]. But a serious consideration of this issue is outside the author's expertise.

IV. CONCLUSIONS

This paper identifies three classes of opportunities for AI researchers at the intersection of quantum computation, quantum theory and AI:

- Design quantum algorithms to solve problems in AI more efficiently;
- Develop more effective methods for formalizing problems in AI by borrowing ideas from quantum theory;
- Develop new AI techniques to deal with problems in the quantum world.

The first class of research is still in the initial stage of development, and not much progress has been made. Shor [37] listed some reasons to explain why quantum algorithms are so hard to discover. Unfortunately, these reasons are valid for the problems in AI too. Some fragmented and disconnected research belonging to the second class have a long history, and some basic ideas can even be traced back to Niels Bohr. In recent years, research in this class has become very active, especially through the *International Symposium on Quantum Interaction* (2007–2009). But it seems that some of these works are quite superficial, and deeper theoretical analysis of the formal methods developed in these works are needed. In particular, more experimental research is required to test the effectiveness. It appears that research in the third class is making steady progress. My main concern is whether the AI techniques developed in this class of research will be useful in quantum physics and will be appreciated by physicists. Certainly, collaboration between AI researchers and physicists will highly benefit the development of this area. Perhaps, experience from bioinformatics can be used for reference where close collaboration between computer scientists and biologists frequently happens and leads to high impact research.

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